## LESSON

## Summer Camp: Penny Plinko Game

## Overview

By definition, the outcome of a random event is impossible to
 know in advance. However, if a random process is repeated often enough, predictable patterns will emerge. This lesson uses a tabletop version of the old carnival game, Plinko to explore patterns in repeated random events.

In Plinko, a round disk drops through a field filled with closely spaced rows of pegs, navigating a zig-zag path to the bottom. As it encounters each peg, the disk has a $50 \%$ possibility of choosing the path to the left or right. By the time the disk has reached the bottom, its final path is an aggregate of many random events - one for each peg it encounters.

This lesson contains design files and instructions to build a Plinko board from laser cut acrylic and wood. Paper fasteners (brads) and pennies constitute the pegs and disks. When each penny drops down the Plinko board, its final position is impossible to predict. However, observing multiple pennies falling, it will become apparent that some outcomes are more likely than others..

The crazy zig-zag motion of a penny wending its bottom of the Plinko board is fun to watch, and makes the board a good basis for games and challenges. The end of this lesson contains ideas for entertaining activities based on Plinko.

THE OBJECTIVE
Observe a repeated random event (penny falling through a Plinko board) to look for patterns in the outcome.

## GRADE LEVEL:

## DIFFICULTY

3-6 Grade

| SUBJECTS | DURATION |
| :--- | :--- |
| Engineering, science, math | 45 Minutes |

## Supplies

- Pennies (a few dozen)


## MATERIALS \& TOOLS:

- Proofgrade Medium Draftboard or other $1 / 8^{\prime \prime}$ wood sheet
- Proofgrade Medium Acrylic sheet
- 67 Standard brads (paper fasteners)
of approximately $3 / 4$ " length (e.g.
https://www.amazon.com/Clipco-Fast
eners-Medium-4-Inch-100-Pack/dp/B
06WD4ZL6P)

Description

## LESSON OUTLINE:

- Build the Plinko Board
- Observe the outcome of repeatedly dropping pennies down the board
- Use the Plink board in activities and games


## Lesson Instructions

## Step 1:

Description


Place the Proofgrade Medium Draftboard on the Glowforge print bed, open the Glowforge App, and click the

Import Artwork button
and then the Upload button

## ©

The piece outlines fill an entire sheet of Draftboard, so check that all pieces fit inside the print area. If the outline of any piece appears black, it won't be printed because it falls outside the printable area. If you select all the pieces as a group, you will be able to move them all around until there are no black lines, ensuring that all pieces will print correctly.

The vector file for the wood pieces contains two different line colors. Because it can be hard to separate very small cut pieces, the grid of very small holes is scored before being cut to ensure they can be removed from the main board. Select the Score print setting for the grid of small holes and place it first in the print order. The rest of the outlines will print on the Cut setting, as shown in the image above. Once the settings are correct, print the wooden pieces. Some of them are very small, so be careful to remove all pieces from the print bed when finished. Peel off any protective paper on the cut pieces.


Remove the Draftboard from the Glowforge and lay a piece of Proofgrade Medium Clear Acrylic Sheet on the print
bed. Click the Import Artwork button and then the Upload button Upload to import the vector file, "PlinkoAcrylicPiece.svg". As with the wood, the small cutouts will be printed twice, first on the Score setting, and then with the rest of the pieces on the Cut setting. Set the Glowforge app print settings accordingly, as shown in the image above. When all settings are correct, print the acrylic cover piece, remove it from the print bed and peel off any protective paper.



Lay the back board and all wooden pieces on a flat surface. Insert the four rails into their corresponding vertical slots in the back board, pressing down until they are firmly seated. Next, insert the eight small T-shaped short pins across the row of holes across the top of the board. When inserted correctly, one edge of the " $T$ " fits snugly into the rectangular slot, while the stem of the "T" extends above the slot to the top edge of the back board. Once the short pins are inserted, slot the edges of the eight long pins into the parallel slots along the bottom of the backboard.


Next, take the two guide pieces and lay them over the rails so they lay flush on top of the back board as shown in the above left image. Once all wooden pieces except the two base pieces are assembled, lay the clear acrylic cover over the backboard. The rails, short pins and long pins will fit snugly into corresponding slots in the acrylic cover. You may have to apply a little pressure to get all pins to fit at once, but don't push too hard or the acrylic may crack.


When the acrylic cover is in place, slip the two semi-circular base pieces upward along the two slots in the bottom edge of the Plinko board. They base pieces contain notches which will lock into place when they are fully inserted and which help secure them to the board. Once the two base pieces are attached, stand the board upright on a flat surface and get the $3 / 4$ " brads which form the "pegs" for the Plinko board.


For each of the small vertical slots in the board, slip a brad stem first into the slot in the acrylic cover and out through the corresponding slot in the wood backboard. When the ends of the brad push through the back, fold them outwards to secure it in place. Repeat this process until all the small vertical slots are filled.


When complete, the Plinko board will stand freely and look like the pictures above.

Step 2: Explorations


Ask students to take some pennies and drop them into the slot at the top of the Plinko board, observing their motion as they fall. At each "peg" (brad) it hits on the way down, the penny will move either to the right or the left. Do the pennies seem to fall more in one direction or the other, or do they zig-zag equally to both sides?

Every time the coin hits a brad, it has a $50 \%$ chance of moving left and a $50 \%$ chance of moving right. As it falls through many rows of pegs, each penny has a chance to travel further and further from its starting position, but it can also travel back towards its origin as well. If you drop a significant number of pennies into the center slot, they will tend to form the "normal" distribution shown in the image above.

Ask the students to examine the completed board. If a penny travels from the top to the bottom, how many opportunities does it have to choose a direction (left or right)? There are ten collisions with pegs or slot dividers along its path to the bottom, so if the board were arbitrarily wide, a penny could end up as far as ten spaces away from where it started. However, this board is only nine columns wide, which affects the shape of the penny distribution at the bottom. Pennies that might normally travel to the "tail" ends of the distribution will be forced back towards the center by the rails. However, it turns out that even if we could make the board wider, only about 4.2\% of pennies dropped from the center would end up beyond the width of this board, so our board size won't have a large effect on the overall distribution of pennies at the bottom.

The path of any penny through the board is a combination of 10 random events, and there is no possible way to predict where any *particular* penny will end up when it reaches the bottom. However, statistics can predict the *probability* of it ending up in any specific location. If the students are advanced enough to understand the concept, you may want to explain to them that probability of a penny's location at the bottom can be viewed as a combination of the choices it has on the way down.

etc...

The image above shows the probability of a penny's location at each of the first three levels on the way down. At the first peg, the penny has a $1 / 2$ (50\%) chance of going left or right, so in level 1 it has a $50 \%$ chance of being in either slot. Moving from level 1 to level 2 , wherever the penny is, it has a $1 / 2$ chance of moving to the left or right. If it moves to the left from the right slot, or to the right from the left slot, it ends up in the center ( $1 / 2$ chance). The other two possible outcomes are to move left from the left slot ( $1 / 4$ chance) or to move right from the right slot $(1 / 4)$ chance. At each level, the probability of the penny ending up in a particular slot is the combination of the possible ways it can arrive there from the previous level.

Statisticians have performed a great deal of work to prove that the most likely outcome for a penny is that it ends up in the column where it started. Students will be able to verify this by experiment.

## Step 3: Evaluate and Test

## Activity 1: Normal Distribution

Ask the students to predict where they think a penny dropped from the center slot is most likely to end up at the bottom of the board. Give students 20 or more pennies, and ask them to drop them all down the center slot one after another. What is the shape of the penny stacks at the bottom? Have them repeat the experiment a number of times. Most of the time, the distribution of pennies will be shaped like a "mountain" with the tallest stack of pennies in the center slot and stacks with progressively decreasing heights nearer the outer edges of the board. Can they explain the outcome qualitatively knowing that at each level a penny has an equal chance to move away from or back towards the center?

## Activity 2: Random Four in a Row

Challenge students to drop pennies down the board until they end up with four in a row across the bottom of the board. Repeating the experiment, ask them to determine whether it's better to drop all pennies in the same slot or to drop them in different slots. How many pennies do they typically have to drop before they have a row of four?

## Activity 3: Reward Chooser/Decision Maker

Use the Plinko board to add an element of fun to decision making. If a student has earned a special reward, place labels across the bottom with each slot specifying a prize. Drop a penny through the board to determine the reward. Alternatively, use labels with different activities to determine what activity to do next or to help determine the outcome of other choices.

